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EDDY DIFFUSIVITY NEAR THE FREE SURFACE OF OPEN CHANNEL FLOW

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Abstract—Eddy diffusivity near the free surface was determined by heat-transfer experiments ($Pr \approx 3$) in broad open channel flow with high heat flux at the surface. It decreases to zero in proportion to Y_s.

A new model is developed, assuming that the damping of turbulence near the surface can be represented by that of the surface wave motion with a wave length equal to the integral scale of turbulence. It represents the experimental result well throughout the cross section. Its limiting relation close to the free surface is well confirmed by gas absorption data of rivers, open channels and liquid films.

NOMENCLATURE

A,	constant in equation (1);
B, C,	constants in equation (5);
c_f,	friction factor;
c_p,	specific heat at constant pressure;
$\mathscr{D}$,	molecular diffusivity;
D,	damping factor for oscillation of flat plate;
D_x, D_y,	damping factors for surface wave;
E,	turbulent energy dissipation;
Fr,	Froude number;
g,	gravitational acceleration;
k,	turbulent kinetic energy;
k_L,	liquid phase mass-transfer coefficient;
M,	mixing constant;
m,	exponent on $\mathscr{D}$ in expression for k_L;
n,	exponent on Y_s in expression for eddy diffusivity;
Pr,	Prandtl number;
Pr_t,	turbulent Prandtl number;
q,	heat flux;
Re,	Reynolds number;
Sc,	Schmidt number;
Sh,	Sherwood number;
T,	time-averaged temperature;
t,	time;
U,	time-averaged velocity in x direction;
u,	fluctuating velocity in x direction;
u^*,	friction velocity;
v,	fluctuating velocity in y direction;
W,	channel width;
We,	Weber number, $u^* \sqrt{(\rho\delta/\sigma)}$;
x,	distance in flow direction;
y,	distance from the floor;
Y_s,	distance from the free surface;
z,	distance mutually perpendicular to x and y.

Greek symbols

α,	wave number;
δ,	flow depth;
ε_D,	eddy diffusivity for mass;
ε_H,	eddy diffusivity for heat;
ε_M,	eddy diffusivity for momentum;
κ,	von Kármán constant;
Λ,	integral scale of turbulence;
λ,	wave length;
μ,	viscosity;
ν,	kinematic viscosity;
ρ,	density;
σ,	surface tension;
ω,	angular velocity;
τ,	shear stress.

Superscripts and subscripts

$^{-}$,	denotes space-averaged variable;
$^{+}$,	denotes dimensionless variable;
s,	indicates free surface condition;
w,	indicates wall condition.

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1. INTRODUCTION

WHILE turbulence mechanisms and turbulent transport of momentum, heat and mass near a solid surface have been extensively studied, little has been reported on the mechanisms in flows near a free gas–liquid interface (free surface). These mechanisms, however, are of great interest from the viewpoint of the decay process of fully turbulent eddies due to the existence of the free surface and play important roles in the problems related to pollution due to waste discharge, gas absorption across the free surface (e.g. reaeration), film condensation and in the field of geophysics.

One of the simplest flows with the free surface is open channel flow. Experimentally, Al-Saffar [1], Jobson and Sayre [2] and several other investigators made extensive investigations on the eddy diffusivity distributions for momentum and mass in open channel flow. As they suggested, in the wall region, turbulent open channel flow is nearly identical with tube flow, but it differs substantially in the outer region, i.e. in the region far from the wall. In the outer region, the eddy diffusivities for momentum, heat and mass decrease with approaching the free surface, whereas those in tube flow have finite values at the centreline. Since the shear stress becomes zero at the free surface and so does the mean velocity gradient, experimental eddy diffusivity results for momentum are not very accurate near the free surface. Because of the experimental condition used by the previous investigators (zero mass flux at the free surface), the same must be said for their results of eddy diffusivity for mass.

Theoretically, Hunt [3] and Ellison [4] proposed eddy diffusivity formulae for open channel flow, based upon von Kármán's hypothesis and simple dimensional arguments, respectively. On the other hand, in the investigation of gas absorption, special attention has been paid to the turbulent flow behaviour in the region very close to the free surface and many theoretical models have been proposed. One is the so-called surface renewal model proposed by Higbie [5] and Danckwerts [6]. In this model, turbulence eddies are assumed to be exposed for a while at the free surface where unsteady molecular diffusion occurs. Other ones are the eddy diffusivity models proposed by Levich [7], Davies [8] and King [9]. The eddy diffusivity distribution is assumed to decrease continuously from a high value in the bulk of the flow to zero at the free surface, and is formulated by dimensional analyses. Because of the lack of available data for the distributions of time-averaged concentration or eddy diffusivity, these models have been tested only by the results of mass-transfer rates across the free surface.

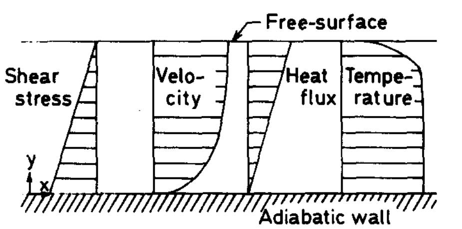

FIG. 1. Schematic diagram of flow and heat transfer.

The purpose of the present research was to determine experimentally the distribution of eddy diffusivity near the free surface. This was accomplished by a heat-transfer experiment at Prandtl numbers of the order of unity. With the heat-transfer condition used in the present work (Fig. 1), heat was removed from the hot water surface into the air with high evaporation rates, and the solid wall (the channel floor) was maintained adiabatic. Thus, in the fully developed condition the vertical heat flux distribution is nearly linear with its maximum at the free surface, and the region where the temperature gradient exists is much thicker than that with a significant concentration gradient in the case of gas absorption across the free surface, the Schmidt numbers being very high. The second purpose of the present study was to deduce a turbulence model to describe the effects of the gravitational force and surface tension acting at the free surface.

2. EXPERIMENTAL APPARATUS

The open channel used is a polyacrylic channel, 49.65-cm wide, 5.5-cm high and 5.00-m long, the floor of which was thermally insulated with 2.0-cm thick polystyrene foam plates.

Measurements were made in the cross sections at which the flow was fully developed (91δ–125δ downstream from the channel entrance). The aspect ratio, W/δ was within 16–38 and the Froude number, $\bar{U}/\sqrt{(g\delta)}$ less than 0.33 where $\bar{U}$ is the mean flow velocity ($\bar{U} < 31$ cm/s) and g the gravitational acceleration. In the heat-transfer experiment water was heated up to 50.0 ± 0.1°C or 58.0 ± 0.1°C in the storage tank and flowed through the open channel. In the open channel heat was removed from the hot water surface to the air by natural convection with high evaporation rates.

Time-averaged velocity and temperature profiles were measured with stagnation pressure and thermocouple probes and a manual traversing equipment. The stagnation pressure probe has a flattened mouth, 0.8×2.5 mm, formed from a 2.0-mm O.D. tubing. The thermocouple probe was a chromel-alumel junction sealed in a 0.25-mm O.D. tube, installed parallel to the flow direction with its tip pointing against the flow.

3. EXPERIMENTAL RESULTS

3.1. *Momentum transfer*

Velocity profiles are normalized with wall parameters and plotted in Fig. 2. Identical profiles at various cross sections confirm that the flow is fully developed.

Measured profiles near the wall are well correlated by van Driest's eddy diffusivity model [10] and suggest that the wall region flow in open channels is identical with that in tubes and flat-plate boundary layers. In the outer part of the wall region, i.e. in the fully turbulent region, the logarithmic law may apply, and the profile is represented by

$$U^+ = (1/\kappa)\ln y^+ + A, \tag{1}$$

where κ is the von Kármán constant, 0.40 and A is 5.75 which is slightly higher than the value of 5.50 for high Reynolds number flow in a tube. Although the accuracy of velocity measurements near the free surface becomes poorer due to the blockage effect of the stagnation pressure probe, the velocity values seem to become higher than equation (1) in approaching the free surface. At the free surface, velocity values obtained by a floating particle method are by 4% higher than equation (1).

If the logarithmic law, equation (1), may apply throughout the water depth, the eddy diffusivity for momentum is given as

$$\varepsilon_M/\nu\delta^+ = \kappa y/\delta(1-y/\delta). \tag{2}$$

The eddy diffusivity distribution calculated from the

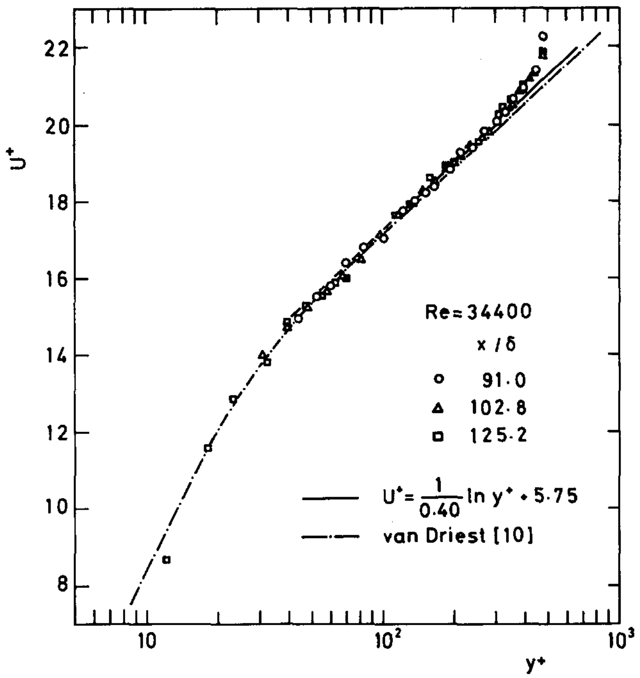

FIG. 2. Distribution of mean velocity in open channel flow.

measured velocity profiles is compared with equation (2) in Fig. 3. The measured distribution has its maximum at $y/\delta = 0.45$ and decreases more rapidly than equation (2) in the outer half region, $y/\delta > 0.45$. There it shows remarkable difference with that in tube flow, e.g. Quarmby and Quirk's result [11].

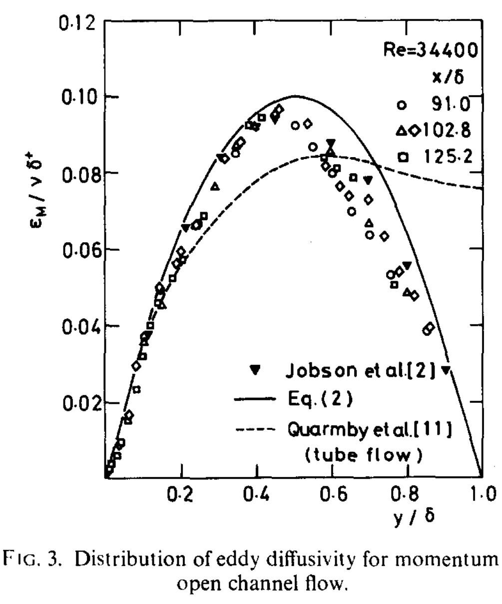

FIG. 3. Distribution of eddy diffusivity for momentum in open channel flow.

3.2. *Heat transfer*

Since the heat transfer above the free surface was caused by natural convection so that the temperature drop of the liquid flow was not so large along the flow direction, it was assumed that the heat transfer occurred with uniform heat flux at the free surface. With the uniform heat flux condition at the free surface and with the adiabatic condition at the floor, the temperature profile under the fully developed condition may be described as

$$T(x, y) = \theta(y) + (q_s/\rho c_p \bar{U}\delta)x. \tag{3}$$

The heat flux distribution is obtained by integrating an energy balance equation as

$$q(y) = \rho c_p \frac{\partial T}{\partial x} \int_0^y U \, dy = (q_s/\bar{U}\delta) \int_0^y U \, dy. \tag{4}$$

The heat flux at the free surface, q_s, was determined from the measured temperature gradients in the flow direction, using equation (3). The longitudinal temperature gradients at various elevations y were confirmed experimentally to agree with each other within 10% and to be constant along the flow direction.

An example of the temperature profiles is shown in Fig. 4. In addition to a steep temperature gradient observed near the free surface,* a detectable temperature gradient is observed even in the bulk of the flow, whereas in the case of mass transfer at high Schmidt numbers the concentration gradient is limited to the region very close to the free surface.

With the temperature profile and with the heat flux obtained from equation (4), the eddy diffusivity for heat ε_H was calculated and plotted in Figs. 5 and 6(a) and (b). The eddy diffusivity distribution for heat is similar in form to that for momentum, having its maximum at $y/\delta = 0.45$ and decreasing steeply in the outer half region, but reveals more drastically in detail the behaviour near the free surface.

Near the free surface it decreases to zero in proportion to the second power of the distance from the

*Even in the region near the free surface, the gradient Richardson number, $-g\,d\rho/dy/\rho(dU/dy)^2$ was larger than -0.01, so that the effect of thermal stratification could be neglected.

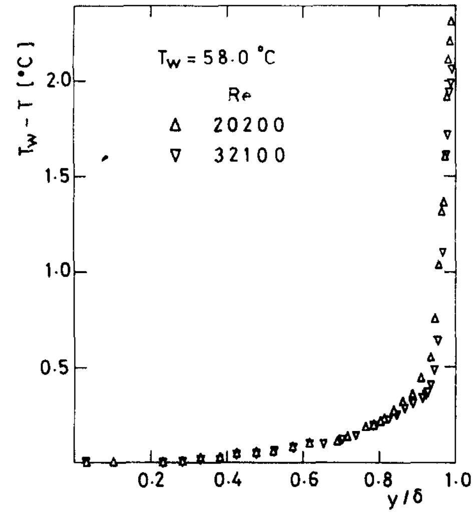

FIG. 4. Typical temperature profiles.

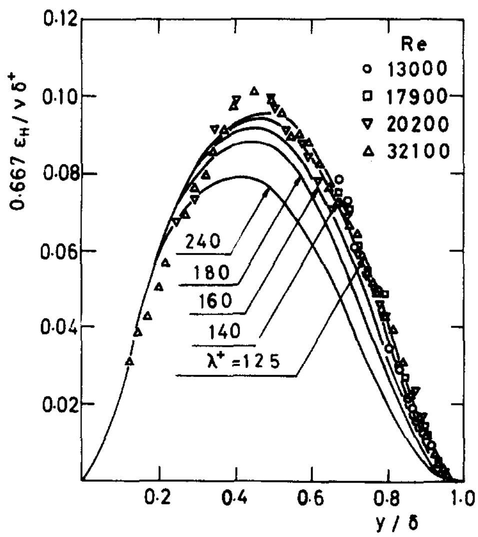

FIG. 5. Distribution of eddy diffusivity for heat in open channel flow.

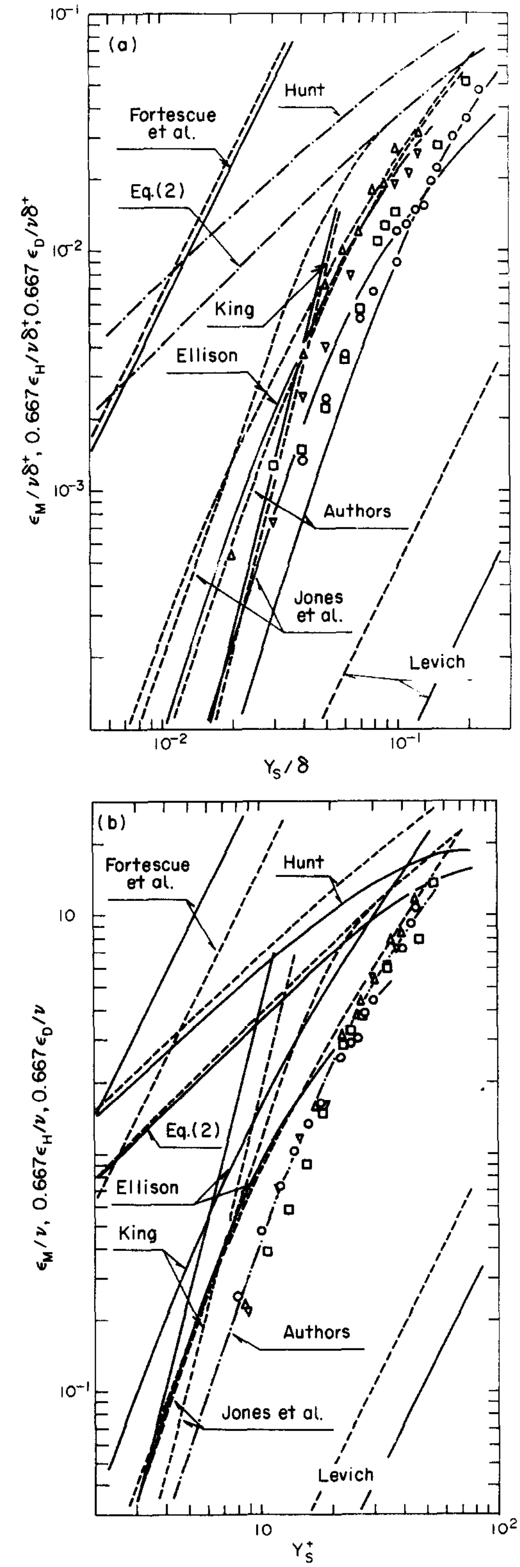

FIG. 6. Eddy diffusivity distributions near the free surface. Symbols same as in Fig. 5. —, $Re = 10^4$; ----, $Re = 3 \times 10^4$; –·–, for all Re.

free surface, but the power seems to increase slightly in the region very close to the free surface. At $Re = 13000$, the minimum value of ε_H/ν detected was 0.396 which was 1.40 times the molecular thermal conductivity and about one eightieth of the maximum value at $y/\delta = 0.45$. In Fig. 6(a) the eddy diffusivity distributions at various Reynolds numbers are plotted in the form of $\varepsilon_H/\nu\delta^+$ vs Y_s/δ where Y_s is the distance from the free surface. Here, some Reynolds number dependence is observed. In Fig. 6(b) these distributions are replotted in the form of ε_H/ν vs Y_s^+. The correlation in this form seems to be better than that in Fig. 6(a).

The distribution of the turbulent Prandtl number Pr_t was obtained from smoothed curves of Figs. 3 and 5 and compared with that in tube flow in Fig. 7. The turbulent Prandtl number is almost constant in the region far from the channel floor and approximately equal to 0.667. The Pr_t distribution agrees well with that of Abbrecht and Churchill [12] for tube flow. Hence, the similarity of Pr_t distributions in the open channel and tube flows will be assumed to hold over the whole flow cross section, so that Pr_t is to be assumed constant and equal to 0.667 also in the region close to the free surface.

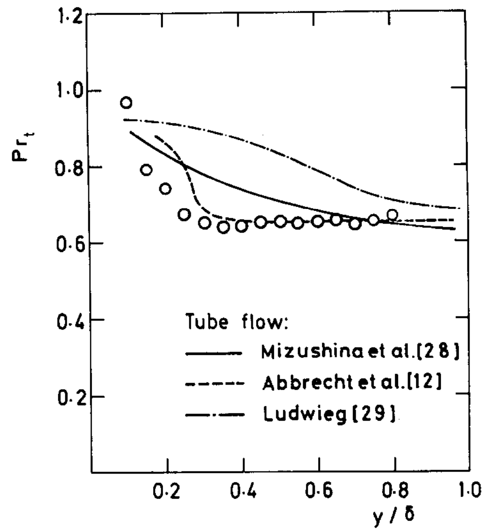

FIG. 7. Turbulent Prandtl number.

4. THEORETICAL CONSIDERATIONS

4.1. *Damping factor model*

Van Driest [10] successfully derived a model to account for the damping of turbulence caused by molecular viscosity action near the solid surface. He assumed that the viscous damping of turbulent fluid oscillation near the solid surface is represented by $(1-D)$, where D is the damping factor of the motion while an infinite flat plate is undergoing simple harmonic oscillations parallel to the plate in an infinite fluid.

Following the same line of physical reasoning, consider the motion of a surface wave at the free surface which is damped in a semi-infinite viscous liquid and assume that the damping of the turbulent fluid oscillations near the free surface can be represented by means of that of the surface wave motion. The motion of a two dimensional surface wave with wave number α ($=2\pi/\lambda$; λ, wave length) in a semi-infinite viscous fluid under the action of gravitational force and surface tension was analysed by Levich [7] and given as

$$u = (i\alpha B\,e^{-\alpha Y_s} - lC\,e^{-lY_s})\,e^{i\alpha x+\omega t}, \tag{5}$$

$$v = (\alpha B\,e^{-\alpha Y_s} + i\alpha C\,e^{-lY_s})\,e^{i\alpha x+\omega t}, \tag{6}$$

where Y_s is the distance from the free surface,

$$C = 2i\nu\alpha^2 B/(\beta+2\nu\alpha^2),\quad l^2 = \alpha^2+\beta/\nu$$

and β is a root of

$$(\beta+2\nu\alpha^2)^2+(\sigma\alpha^3/\rho+g\alpha) = 4\nu^2\alpha^4\sqrt{(\beta/\nu\alpha^2+1)}. \tag{7}$$

These expressions may approximately hold even if the liquid layer has a finite depth which is almost equal to the wave length. In addition, these expressions may apply to three dimensional waves, if in the place of α one puts $\sqrt{(\alpha_x^2+\alpha_z^2)}$. If $\alpha_z \gg \alpha_x$, α may be approximated by α_z, so that λ represents λ_z. Damping factors of the motion, i.e. amplitude ratios of the x and y components of the motion to those at the free surface are given as

$$D_x = |i\alpha\,e^{-\alpha Y_s} - lC/B\,e^{-lY_s}|/|i\alpha - lC/B|, \tag{8}$$

$$D_y = |e^{-\alpha Y_s} + iC/B\,e^{-lY_s}|/|1+iC/B|. \tag{9}$$

On the other hand, assume that the fully turbulent flow in an open channel is described by equation (1), if effects of the gravitational force and surface tension acting at the free surface are to be ignored. Then, after introducing the damping factors into each velocity component in the Reynolds stress term, the eddy diffusivity for momentum is to be expressed as

$$\varepsilon_M/\nu\delta^+ = \kappa y/\delta(1-y/\delta)(1-D_x)(1-D_y). \tag{10}$$

To take into account the damping of turbulence due to the channel floor, van Driest's damping factor D will be used in the following form;

$$\varepsilon_M/\nu\delta^+ = \kappa y/\delta(1-y/\delta)(1-D_x)(1-D_y)(1-D)^2, \tag{11}$$

where D is taken to be equal to $\exp(-y^+/26)$. The above modification for the viscosity effect near the wall is different from the eddy diffusivity model originally derived by van Driest. Near the wall, however, equation (11) decreases in proportion to $(y^+)^3$ and represents the experimental results of many investigators very well.

The $\varepsilon_M/\nu\delta^+$ distribution was calculated from the present model with various values of $\lambda^+(=\lambda u^*/\nu)$, and compared with the experimental results in Fig. 8. With

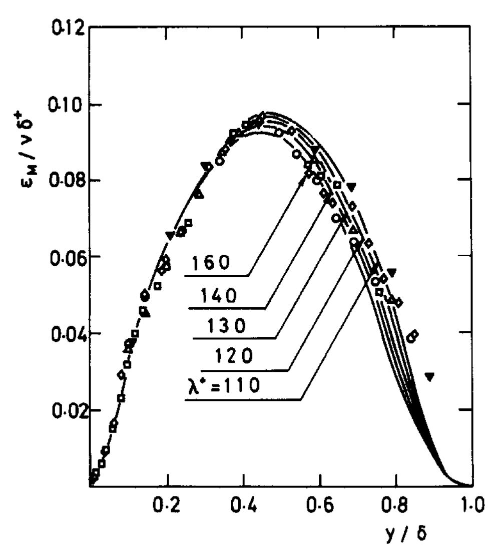

FIG. 8. Distribution of eddy diffusivity for momentum in open channel flow. Symbols same as in Fig. 3.

the constant value of Pr_t equal to 0.667, the predicted distribution of eddy diffusivity for heat is shown in Figs. 5 and 6(a) and (b). In these figures, the best fit with the experimental results is obtained if the characteristic wave length λ^+ is taken to be 140. It is worth noting that the present model fits well over the whole cross section including the region near the free surface.

The value $\lambda^+ = 140$ for the characteristic wave length in the present model is of the same order of magnitude as integral scales of turbulence. Since the spanwise integral scale Λ_z^+ is about one fifth of the longitudinal one, Λ_x^+, which is about 600 throughout the cross section from the measurements of McQuivey *et al.* [13], λ^+ may be regarded as representing the spanwise integral scale. Hence, it may be said that the behaviour of the eddy diffusivity near the free surface can well be explained by considering the damping of energy containing eddies due to the gravitational force and surface tension at the free surface.

The friction factor predicted by the present model is shown in Fig. 9. Good agreement is seen with Blasius' [14] and Ismail's [15] formulae for low and high Reynolds number regions, respectively.

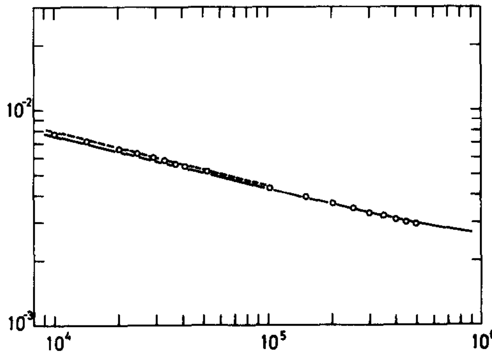

FIG. 9. Change of friction factor with Reynolds number in open channel flow. ○, Authors' model for $\lambda^+ = 140$; ---, Blasius [14], $c_f = 0.0791 Re^{-0.25}$; —, Ismail [15], $1/[2\sqrt{(c_f)}] = -1.32 + 0.95 \times \ln[2Re\sqrt{(c_f)}]$.

4.2. *Eddy diffusivity near the free surface*

Near the free surface the eddy diffusivity expression (11) of the present model may be well approximated by

$$\varepsilon_M/\nu\delta^+ = \kappa Y_s^+/\delta^+(1 - Y_s^+/\delta^+) \times [1 - \exp(-Y_s^+/30.4)]^2. \quad (12)$$

The limiting relation of the above equation is

$$\varepsilon_M/\nu = 4.331 \times 10^{-4}(Y_s^+)^3 \text{ for } Y_s^+ \to 0. \quad (13)$$

Several investigations have been done on the eddy diffusivity near the free surface. Al-Saffar [1] and Jobson and Sayre [2] recommended equation (2) resulting from the logarithmic velocity profile. From an application of von Kármán's hypothesis to the open channel flow, Hunt [3] recommended

$$\varepsilon_M/\nu\delta^+ = 2\kappa Y_s/\delta[1 - \sqrt{(Y_s/\delta)}]. \quad (14)$$

Ellison [4] assumed that the components of the turbulence which contributed to the eddy diffusivity near the free surface were determined entirely by local quantities, i.e. by the local shear stress and the distance from the surface. Hence, employing the usual dimensional arguments he proposed an equation for infinite Reynolds number

$$\varepsilon_M/\nu\delta^+ = M(Y_s/\delta)^{1.5}, \quad (15)$$

and another for finite Reynolds numbers

$$\varepsilon_M/\nu\delta^+ = \{[1 + (2M\delta^+)^2(Y_s/\delta)^3]^{1/2} - 1\}/2\delta^+ \approx (M^2/\delta^+)(Y_s^+)^3 \text{ for } Y_s \to 0, \quad (16)$$

where M was referred to as the mixing constant at the free surface which was determined from apparent longitudinal diffusion experiments as 0.8. From simple dimensional arguments based on Prandtl's mixing length concept, the following expression has been proposed by Levich [7] and supported by Davies [8]:

$$\varepsilon_M = (0.4\rho u^*/\sigma) Y_s^2, \quad (17)$$

where σ is the surface tension.

Examining experimental results of gas absorption into falling liquid films and assuming that the eddy diffusivity is related to the smaller eddies, King [9] derived the following empirical formula:

$$\varepsilon_D/\nu\delta^+ = 0.006(\bar{E}\rho/\mu^2) Y_s^4/\nu\delta^+ = 0.006[Re/(\delta^+)^3](Y_s^+)^4/4. \quad (18)$$

Based on the surface renewal concept, Fortescue and Pearson [16] considered that roll cells with their size equal to the integral scale of turbulence were aligned at the free surface. Analysis of mass transfer into the roll cells resulted in

$$k_L = 1.46\mathscr{D}^{1/2}(u'/\Lambda_x)^{1/2}. \quad (19)$$

If $u' = 0.1\,\bar{U}$ and $\Lambda_x = 0.1\delta$ as they assumed for the flow in rivers, the eddy diffusivity for mass is given as

$$\varepsilon_D/\nu\delta^+ = 5.26(\bar{U}/u^*)(Y_s/\delta)^2. \quad (20)$$

Recently, Jones and Launder [17, 18] devised a turbulence model in which the eddy diffusivity was determined from the solutions of transport equations for the turbulent kinetic energy k and the energy dissipation E. For the transport equations suitable forms were provided to take into account the case of low Reynolds number of turbulence. This model was applied to open channel flow with the boundary conditions, $k = E = 0$ both at the floor and at the free surface.

Comparisons of the above-mentioned expressions with the experimental results are shown in Figs. 6(a) and (b). The eddy diffusivities are shown in the form of $\varepsilon_{H,D}/\nu\delta^+$ vs Y_s/δ in Fig. 6(a) and in the form of $\varepsilon_{H,D}/\nu$ vs Y_s^+ in Fig. 6(b). The discrepancy between these expressions is surprising; it is mainly due to the differences in the exponent on Y_s^+ and the Reynolds number dependence in their limiting behaviours close to the free surface. The limiting relations for the eddy diffusivity resulting from the logarithmic profile and for Hunt's model give the exponent of $n = 1$. The exponent n is 1.5 in Ellison's model for infinite Reynolds number, 2 in the models of Levich and Fortescue and Pearson and 4 in King's model. The present model, Jones and Launder's model and Ellison's model for finite Reynolds number predict an exponent of $n = 3$. The experimental results presented here demonstrate that the exponent n is larger than 2.

Information about the exponent n is given from a knowledge of the variation of gas absorption rates with the molecular diffusivity $\mathscr{D}$. If the eddy diffusivity is expressed as $\varepsilon_D = aY_s^n$ in the region where concentration gradients exist, the mass-transfer coefficient is approximately given by

$$k_L = 1/\int_0^\infty dY_s/(\mathscr{D} + aY_s^n), \quad (21)$$

which may be integrated to give

$$k_L = (n/\pi)a^{1/n}\mathscr{D}^{1-1/n}\sin(\pi/n). \quad (22)$$

Table 1. Dependence of mass transfer across the free surface on diffusivity $\mathscr{D}$

Authors	m	
Kozinsky *et al.* [19]	0.52–0.60	Low stirring speed
	0.69–0.78	High stirring speed
Davies *et al.* [21]	0.59–0.63	Uniform-rotation
	0.65–0.71	Counter-rotation
Dobbins [20]	0.50–0.73	Oscillating lattice below water surface

Thus, the values of n equal to 2, 3 and 4 require the variation of k_L with $\mathscr{D}$ to the power of m equal to 1/2, 2/3 and 3/4, respectively. The power m was determined experimentally by Kozinski and King [19], Dobbins [20] and Davies, Kilner and Ratcliff [21] the results of which are listed in Table 1. The power m seems to be larger than 1/2 and so n is larger than 2. The upper limit of m is thought to be smaller than 0.7, since a high value of m is due to a high stirring speed which is accompanied by bubbling and rippling and/or due to artificial agitation just beneath the free surface which causes turbulence energy production there and so yields conditions which are different from ordinary turbulence behaviour near the free surface. Thus, it may be reasonably accepted that the limiting relation of the eddy diffusivity close to the free surface is proportional to the third power of Y_s. It should be noted, however, that the m-value becomes smaller than 2/3 for such low Reynolds number flows as falling liquid films. In this case the exponent n gradually becomes smaller with increasing Y_s within the region where the concentration gradients exist.

Of the three models which have the exponent n equal to three, the authors' and Jones and Launder's models represent the experimental eddy diffusivity better than Ellison's model for finite Reynolds numbers. The Reynolds number dependence is also different between these models. In the authors' and Jones and Launder's models the eddy diffusivity normalized with the kinematic viscosity can be approximated by a universal function of Y_s^+, but in Ellison's model it decreases with increasing Re, and thus increasing δ^+. Experimentally the independence of the relation $\varepsilon_M/\nu = fn(Y_s^+)$ of Reynolds number is evidenced in the comparison of Figs. 6(a) and (b). This will also be confirmed in the following section where reaeration across the free surface of rivers, open channels and falling liquid films is discussed for a Reynolds number range from 2.6×10^3 to 1.3×10^7.

4.3. *Reaeration of rivers, open channels and falling liquid films*

Experimental information concerning reaeration across the free surface is important for gas absorption operations in chemical industries and the environmental protection against water pollution. It is also important as the only information for testing the eddy diffusivity models very close to the free surface. Experimental investigations cited here were those of O'Connor and Dobbins [22] and Churchill *et al.* [23] for rivers, of Streeter *et al.* [24] and Krenkel and Orlob [25] for open channel flow and of Davies and Warner [26] for vertical and inclined falling liquid film flows. The Reynolds number ranges from 2.6×10^3 to 1.3×10^7 and the water depth is from 0.057 to 347 cm as listed in Table 2.

The models of Fortescue and Pearson ($n = 2$), Levich ($n = 2$), the authors ($n = 3$), Ellison ($n = 3$) and King ($n = 4$) were tested with those experimental informations. The expressions for the mass-transfer coefficient derived from these models are shown in Table 3. Comparisons of these expressions with the experimental results are shown in Figs. 10(a)–(e).

Before examining Figs. 10(a)–(e), it should be noted that rivers are highly contaminated with surface active agents so that the experimental mass-transfer coefficients are probably about half of those for clean streams or less, as suggested by Downing *et al.* [27] and Davies [8]. The data for falling liquid films, especially at high Reynolds numbers, might be highly complicated due to the interaction between gas and liquid flows and are thought to be overestimated because the mass transfer surface is increased by the rippling of the free surface. The data of Streeter *et al.* which were obtained in recirculation channels must be considered to be too large because much of the oxygen transfer has probably taken place at the point of energy

Table 2. Experimental conditions

Authors	δ (cm)	$Re \times 10^{-4}$	Sc	
O'Connor *et al.* [22]	27–280	2.6–630	337–739	Rivers (Elk, Clarion, Brandywine, Tennessee, Illinois), San Diego Bay
Churchill *et al.* [23]	65–347	176–1328	352–805	Rivers (Clinch, Holston, Fr. Broad, Watauga, Hiwassee)
Krenkel *et al.* [25]	2.4–6.1	0.92–1.36	414–659	Open channel (18 in × 1 ft × 60 ft)
Streeter *et al.* [24]	9.65, 30.5	0.33–8.1	355–740	Recirculation channels (3- and 6-in wide)
Davies *et al.* [26]	0.057–0.103	0.26–0.50	444	Falling liquid films: vertical and inclined (24°56′)

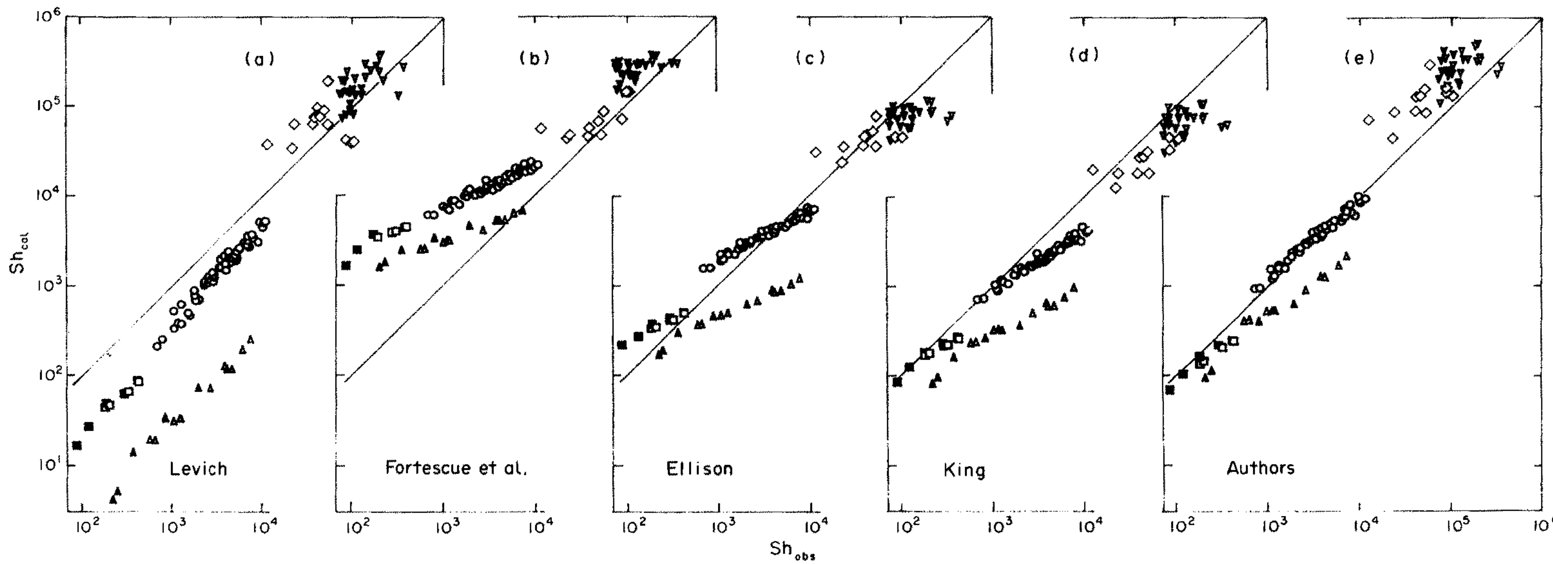

FIG. 10. Comparison between predicted and observed mass-transfer coefficients across the free surface. ○, Krenkel *et al.* [25], open channel: △, ▲, Streeter *et al.* [24], recirculation channels: ▽, Churchill *et al.* [23], rivers: ◇, O'Connor *et al.* [22], rivers: □, Davies *et al.* [26], falling liquid films: ■, Davies *et al.* [26], liquid films on inclined plate.

Table 3. Expressions for mass-transfer coefficient and Sherwood number

Models	n	Expressions for k_L and Sh
O'Connor and Dobbins [22]	2	$k_L = (\mathscr{D}\bar{U}/\delta)^{1/2}$, $Sh = 2.00(Re \cdot Sc)^{1/2}$
Fortescue and Pearson [16]	2	$k_L = 1.46(\mathscr{D}\bar{U}/\delta)^{1/2}$, $Sh = 2.92(Re \cdot Sc)^{1/2}$
Levich [7] and Davies [8]	2	$k_L = 0.35[\mathscr{D}(u^*)^3\rho/\sigma]^{1/2}$, $Sh = 1.40(\delta^+ \cdot Sc)^{1/2} We$
Ellison [4]	3	$k_L = 0.879[(\mathscr{D}u^*)^2/\nu\delta]^{1/3}$, $Sh = 3.26(\delta^+)^{2/3} Sc^{1/2}$
Authors	3	$k_L = 0.0626 u^* Sc^{-2/3}$, $Sh = 0.250\delta^+ Sc^{1/3}$
King [9]	4	$k_L = 0.250(\mathscr{D}^3 \bar{E}\rho/\mu^2)^{1/4}$, $Sh = (\bar{E}\rho^2\delta^4/\mu^3)^{1/4} Sc^{1/4}$

input into the channels. Therefore, Krenkel and Orlob's experiment made in a broad open channel gives the most reliable information.

Figures 10(a) and (b) show the results from the models of Levich, and Fortescue and Pearson, respectively, with $n = 2$ compared with the experimental results. Although they hold well in rivers, they predict unconceivable values for the mass-transfer coefficient in falling liquid films. Therefore, these models are excluded from further considerations. Of the remaining three models with $n = 3$ and 4, Ellison's model ($n = 3$) might be thought to correlate best the results of rivers as well as those of open channels. However, a detailed examination shows that almost all the predictions from Ellison's model lie below the experimental results in rivers, and King's model gives even smaller values. If the effect of surface active agents on reducing the absorption rates is considered, these models must be suspected to give mass-transfer values which are by up to a factor of 2 or smaller than those for clean water streams.

Predictions based on the present model show an excellent agreement with Krenkel and Orlob's results and agree well with the results of inclined liquid films. Agreement with the river data is also good if 50% reduction of the absorption rates from those for clean water streams is accepted.

5. CONCLUSIONS

The effects of the gravitational force and surface tension acting on the free surface have been found to be significant for the distribution of eddy diffusivities in the outer half of the flow cross section in a broad, straight open channel. Especially in the region close to the free surface, the eddy diffusivity was confirmed to decrease to zero in proportion to the distance from the free surface to the power of more than two.

These effects can satisfactorily well be explained by a model based on the assumption that the damping of turbulent fluid oscillations near the free surface can be represented by that of the surface wave motion in a viscous semi-infinite fluid with a wave length of the same magnitude as the integral scale of turbulence. The proposed limiting relation for the eddy diffusivity close to the free surface was well confirmed by the experimental data for the reaeration of turbulent liquid streams, i.e. in falling liquid films, open channels and rivers in the Reynolds number range from 2.6×10^3 to 1.3×10^7.

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DIFFUSIVITE TURBULENTE PRES DE LA SURFACE LIBRE D'UN ECOULEMENT EN CANAL OUVERT

Résumé—La diffusivité thermique à proximité de la surface libre a été déterminée par des expériences de transfert thermique ($Pr = 3$) dans un écoulement en canal large et ouvert avec un flux thermique élevé à la surface. Elle décroit jusqu'à zéro proportionnellement à Y^3. On donne un nouveau modèle qui suppose que l'amortissement de la turbulence près de la surface peut être représenté par un mouvement de l'onde de surface avec une longueur d'onde égale à l'échelle intégrale de turbulence. Il représente bien le résultat expérimental dans la section droite. La relation limite près de la surface libre est bien confirmée par l'absorption de gaz pour les rivières, les canaux ouverts et les films liquides.

DER SCHEINBARE DIFFUSIONSKOEFFIZIENT IN DER NÄHE DER FREIEN OBERFLÄCHE EINER EFFENEN KANALSTRÖMUNG

Zusammenfassung—Der scheinbare Diffusionskoeffizient in der Nähe der freien Oberfläche wurde mit Hilfe von Wärmeübergangs-Experimenten ($Pr = 3$) in einer breiten, offenen Kanalströmung mit hoher Wärmestromdichte an der Oberfläche bestimmt. Es ergab sich eine Y^3 proportionale Abnahme bis auf Null.

Es wird ein neues Modell entwickelt, wobei davon ausgegangen wird, daß die Dämpfung der Turbulenz in Oberflächennähe durch die Dämpfung der Oberflächenwellen, deren Wellenlänge dem integralen Längenmaß der Turbulenz entspricht, wiedergegeben werden kann. Damit können die Meßwerte im gesamten Querschnitt gut wiedergegeben werden. Die sich in der Nähe der freien Oberfläche ergebenden Grenzwerte werden durch Gasabsorptionsdaten für Flüsse, offene Kanäle und Flüssigkeitsfilme bestätigt.

ТУРБУЛЕНТНАЯ ТЕМПЕРАТУРОПРОВОДНОСТЬ ОКОЛО СВОБОДНОЙ ПОВЕРХНОСТИ ПОТОКА В ОТКРЫТОМ КАНАЛЕ

Аннотация — В опытах по переносу тепла при $Pr \approx 3$ определялся коэффициент турбулентной температуропроводности у свободной поверхности потока в широком открытом канале при большой плотности теплового потока у поверхности. Значение коэффициента уменьшается как y^3. Разработана новая модель, предполагающая, что затухание турбулентности у поверхности может быть представлено как затухание волнового движения поверхности с длиной волны, равной интегральному масштабу турбулентности. Модель хорошо описывает экспериментальные данные по всему поперечному сечению. Следующее из модели предельное соотношение, справедливое вблизи свободной поверхности, подтверждается данными по адсорбции газа поверхностью рек, течением в открытых каналах и жидких пленках.